Large Deviations for Empirical Measures Arising in Importance Samplings

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Importance sampling is a popular method for efficient computation of various properties of a distribution such as probabilities, expectations, quantiles, etc. In particular for probabilities for extreme values, high quantiles and other risk measures that depend on the tail of the distribution. The output of an importance sampling algorithm can be represented as a weighted empirical measure, where the weights are given by the likelihood ratio between be original distribution and the sampling distribution. In this paper the efficiency of an importance sampling algorithm is studied by means of the rate function associated to the large deviations for the weighted empirical measure. The main result characterizes the large deviations of the weighted empirical measure away from the true underlying distribution. The result, which is stated as a Laplace principle for the weighted empirical measure arising in importance sampling, can be viewed as a weighted version of Sanov's theorem. The main theorem is applied to quantify the performance of an importance sampling algorithm over a collection of subsets of a given target set as well as quantile estimates. The proof of the main theorem relies on the weak convergence approach to large deviations developed by Dupuis and Ellis.

Key Words: Large deviations, Importance sampling, Extreme values, Weak convergence