Optimal filtering and the dual process

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We link optimal filtering for hidden Markov models to the notion of duality for Markov processes. We show that when the signal is dual to a process that has two components, one deterministic and one a pure death process, and with respect to functions that define changes of measure conjugate to the emission density, the filtering distributions evolve in the family of finite mixtures of such measures and the filter can be computed at a cost that is polynomial in the number of observations. The result are applied to some known cases, where the signal is the Cox-Ingersoll-Ross diffusion or the one-dimensional Wright-Fisher diffusion, and to new cases, such as the K-dimensional Wright-Fisher diffusion and the Fleming-Viot process with parent independent mutation.

Keywords: Bayesian conjugacy, Dirichlet process, Finite mixture models, Hidden Markov model