

## Shape Constraints in Empirical Bayes Inference

Ivan Mizera\*

University of Alberta, Edmonton, Canada [imizera@yahoo.com](mailto:imizera@yahoo.com)

Mu Lin

University of Alberta, Edmonton, Canada [lm@ualberta.ca](mailto:lm@ualberta.ca)

Following the earlier work on the estimation of densities for the classical Gaussian compound decision problem and their associated (empirical) Bayes rules, the problem that has been considered from several perspectives—introducing a nonparametric maximum likelihood estimator of the mixture density subject to a monotonicity constraint on the resulting Bayes rule, and the Kiefer-Wolfowitz nonparametric maximum likelihood estimator for mixtures—we propose modifications that require a shape-constraint of log- or quasi-concave type on the mixture density. These modifications exhibit superior behavior in the case when the shape assumption reasonably captures the behavior of the data, in particular, when the mixing distribution is unimodal. Like the earlier approaches, our proposals are also cast as convex optimization problems, and can be efficiently solved by modern convex optimization methods. The finite-sample properties are compared with several existing empirical Bayes and other methods in a small simulation study.

**Key Words:** Empirical Bayes, Shape Constrained Inference, Nonparametric Maximum Likelihood, Shrinkage, Compound Decision Problem, Mixture Model, Random Effects, Convex Optimization.